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Edge Product Cordial Labeling of Splitting Graphs

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Abstract

Let G=(V,E) be a graph. The splitting graph of G is obtained by adding a new vertex u' corresponding to each vertex u of G such that $N(u)=N(u')$ and is denoted by $S'(G)$. An edge labeling function $f : E(G) \rightarrow \{0,1\}$ with the induced vertex labeling $f^* : V(G) \rightarrow \{0,1\}$ defined as $f^*(v) = \prod f(e_i)$ where the product is taken over all e_i incident with v is called an edge product cordial labeling of G if $| v_f(0) - v_f(1) | \leq 1$ and $| e_f(0) - e_f(1) | \leq 1$. A graph G is called an edge product cordial graph if it admits edge product cordial labeling. In this paper, we find the edge product cordial labeling of splitting graph of some standard and special graphs.

Keywords: Splitting graph, Edge product cordial labeling.

AMS subject classification: 05C78

1. Introduction

Let G = (V, E) be a finite, undirected graph without loops and multiple edges. If the vertices of the graph are assigned values subject to certain conditions, then it is known as graph labeling. For extensive survey on graph labeling, refer Gallian, J.A.[2]. A mapping $f : V(G) \rightarrow \{0,1\}$ is called binary vertex labeling of G and f(v) is called the label of vertex v of G under f. The concept of cordial labeling was introduced by Cahit, I.[1]. A binary vertex labeling of graph G with induced edge labeling $f^* : E(G) \rightarrow \{0,1\}$ defined by $f^* (e = uv) = f(u) f(v)$ is called a product cordial labeling if $| v_f(0) - v_f(1) | \leq 1$ and $| e_f(0) - e_f(1) | \leq 1$. A graph G is product cordial if it admits product cordial labeling. A binary edge labeling of graph G with induced vertex labeling $f^*: V(G) \rightarrow \{0,1\}$ defined by $f^*(v) = \prod f(e_i)$ for$\{e_i \in E(G) / e_i\text{'s incident to } v\}$ is called an edge product cordial labeling if $| v_f(0) - v_f(1) | \leq 1$ and $| e_f(0) - e_f(1) | \leq 1$. A graph G is edge product cordial if it admits edge product cordial labeling. The splitting graph of G is obtained by adding a new vertex u' corresponding to each vertex u of G such that $N(u) = N(u')$ and is denoted by S'(G). For $n \geq 2$, $P_n \odot K_1$ is called a comb graph. In this paper, we analyze splitting graphs of P_n, C_n and $P_n \odot K_1$ for the existence of edge product cordial labeling.

2. Edge Product Cordial Labeling Of Splitting Graphs

Theorem 2.1: $S(P_n)$ is an edge product cordial graph if and only if n is even.

Proof: $S(P_n)$ has 2n vertices and 3n – 3 edges.

Let $V(S(P_n)) = \{v_1,v_2,\ldots,v_n,u_1,u_2,\ldots u_n\}$ with $(v_1,v_2,\ldots,v_n)$ as the base path and u_i as the vertex corresponding to v_i.

Case 1: n is even

The following type of labeling gives maximum value for $v_f(1)$.

Define $f : E(S(P_n)) \rightarrow \{0,1\}$ as follows.

$f(v_i v_{i+1}) = 1$ for $i = 1$ to $\frac{n}{2} - 1$; $f(v_i u_{i+1}) = 1$ for $i = 1$ to $\frac{n}{2} - 1$ and $f(v_{n-1} u_n) = 1$

$f(u_i v_{i+1}) = 1$ for $i = 1$ to $\frac{n}{2}$; $f(uv) = 0$ otherwise.

Correspondingly,

$f(v_i) = 1$ for $i = 1$ to $\frac{n}{2} - 1$; $f(u_i) = 1$ for $i = 1$ to $\frac{n}{2}$ and $f(u_n) = 1$; $f(v) = 0$ otherwise.

Therefore $v_f(1) = \frac{n}{2} - 1 + \frac{n}{2} + 1 = n - 1+1 = n$ $\longrightarrow$ I

$v_f(0) = 2n - n = n$ $\longrightarrow$ II

$e_f(1) = \frac{n}{2} - 1 + \frac{n}{2} + \frac{n}{2} = \frac{3n}{2} - 1$ $\longrightarrow$ III

$e_f(0) = (3n - 3) - (\frac{3n}{2} - 1) = 3n - 3 - \frac{3n}{2} + 1$

$= \frac{3n}{2} - 2$ $\longrightarrow$ IV

From I and II, $|v_f(0) - v_f(1)| = |n - n| = 0 \leq 1$

From III and IV, $|e_f(0) - e_f(1)| = \left|\frac{3n}{2} - 2 - (\frac{3n}{2} - 1)\right| = 1 \leq 1.$

Therefore f is an edge product cordial labeling of $S(P_n)$.

Thus, $S(P_n)$ is an edge product cordial graph if n is even.

Case 2: n is odd

Number of edges = 3n – 3 = 3(n – 1) = even

Therefore, in any edge labeling there must be equal number of 0's and 1's for edges.

(i.e.) $e_f(0) - e_f(1) = \frac{3(n-1)}{2}$

Given below is one such edge labeling which assigns 1 for maximum number of vertices.

$f(v_i v_{i+1}) = 1$ for $i = 1$ to $\left[\frac{n}{2}\right]$; $f(u_i v_{i+1}) = f(v_i u_{i+1}) = 1$ for $i = 1$ to $\left[\frac{n}{2}\right]$ and $f(e) = 0$ otherwise.

Correspondingly,

$f(v_i) = f(u_i) = 1$ for i = 1 to $\left[\frac{n}{2}\right]$ and $f(v) = 0$ otherwise

Therefore, $v_f(1) = \left[\frac{n}{2}\right] + \left[\frac{n}{2}\right] = n - 1$ (since n is odd)

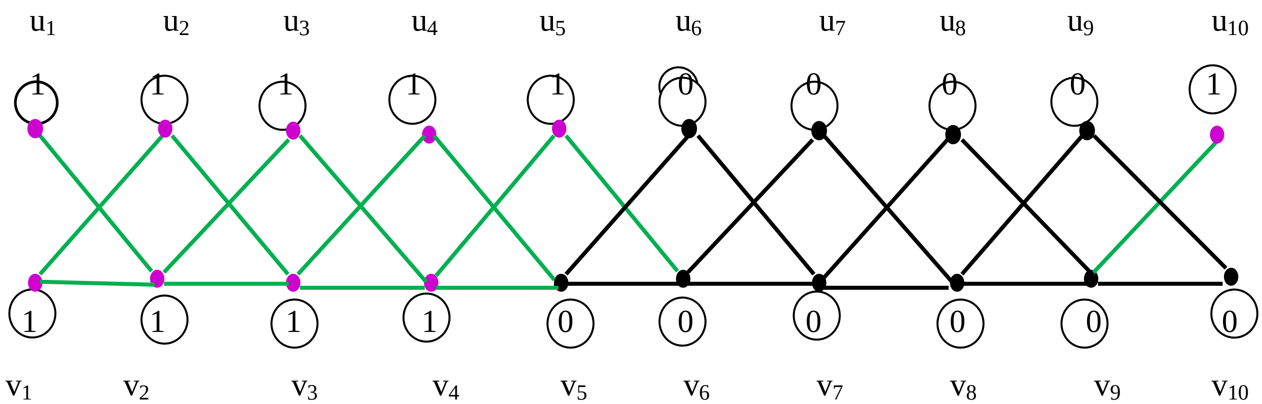
$v_f(0) = 2n - \overline{n-1} = n + 1$

$\therefore |v_f(0) - v_f(1)| = |n - 1 - \overline{n+1}| = 2 > 1.$

Therefore, the above labeling is not edge product cordial. Since this type of labeling gives maximum number of possible 1's to the edges, no other edge labeling can be edge product cordial.

$\therefore$ $S(P_n)$ is not an edge product cordial graph if n is odd. Hence, by cases 1 and 2, $S(P_n)$ is an edge product cordial graph if and only if n is even.

Illustration 2.2 : $S(P_{10})$ is an edge product cordial graph.



$S(P_n)$ has 2n vertices and 3n – 3 edges.

Here, In $S(P_{10})$, n = 10 (even). The light and dark edges represents the edges with labels 1 and 0 respectively.

From the figure, it is clear that

$e_f(1) = 4 + 5 + 4 + 1 = 14$ and $e_f(0) = 5 + 4 + 4 = 13$

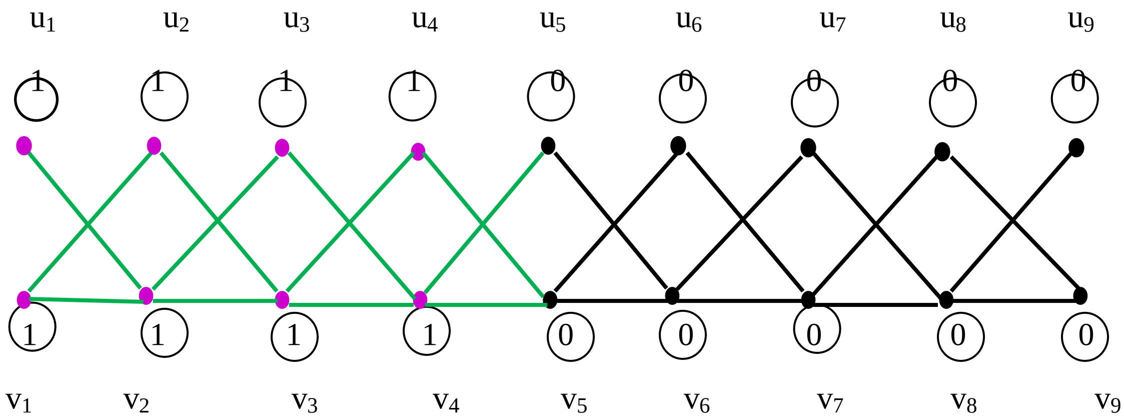
Therefore, $| e_f(0) - e_f(1) | = |13 - 14| = 1 \leq 1$

$v_f(1) = 4 + 5 + 1 = 10$ and $v_f(0) = 6 + 4 = 10$

Thus, $| v_f(0) - v_f(1) | = |10 - 10| = 0 \leq 1.$

Hence, $S(P_{10})$ is an edge product cordial graph.

Illustration 2.3: $S(P_9)$ is not an edge product cordial graph.



$S(P_n)$ has 2n vertices and 3n – 3 edges.

Here, In $S(P_9)$, n = 9 (odd). The light and dark edges represents the edges with labels 1 and 0 respectively.

The following type of labeling gives maximum number for $v_f(1)$. From the figure, it is clear that

$e_f(1) = 4 + 4 + 4 = 12$ and $e_f(0) = 4 + 4 + 4 = 12$

Therefore, $| e_f(0) - e_f(1) | = |12 - 12| = 0$

$v_f(1) = 4 + 4 = 8$ and $v_f(0) = 5 + 5 = 10$

Thus, $| v_f(0) - v_f(1) | = |10 - 8| = 2 > 1$

Therefore, the above labeling is not an edge product cordial. Since this type of labeling gives maximum number of possible 1's to the edges, no other edge labeling can be edge product cordial.

Hence, $S(P_9)$ is not an edge product cordial graph.

Theorem 2.4 : $S(C_n)$ is not an edge product cordial graph for $n \geq 3$.

Proof: $S(C_n)$ has 2n vertices and 3n edges.

Let $V(S(C_n)) = \{v_i , u_i / i = 1 \text{ to } n\}$ with $(v_1,v_2,\ldots\ldots\ldots,v_n)$ as the base cycle and u_i's as the corresponding vertices.

The following type of labeling gives maximum number for $v_f(1)$.

In any edge product cordial labeling of $S(C_n)$, $| e_f(0) - e_f(1) | = 0$

$$\therefore e_f(0) = e_f(1) = \frac{3n}{2}$$

Among the various edge product cordial labelings, $v_f(1)$ is maximum only when f(e) = 1 for e's incident with u_i .

Since the number of edges incident with each u_i is 2 and are distinct, $v_f(1) \leq \frac{e_f(1)}{2} = \frac{3n}{4}$

$v_f(0) \geq 2n - \frac{3n}{4} = \frac{8n-3n}{4} = \frac{5n}{4}$

$\therefore$ | $v_f(0)$ - $v_f(1)$ | $\geq \left|\frac{5n}{4} - \frac{3n}{4}\right| = \frac{2n}{4}$

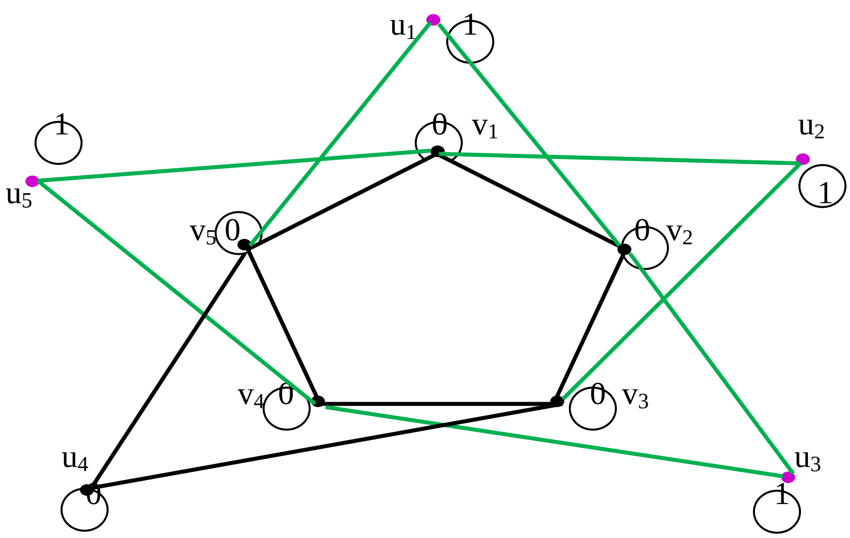
$\therefore$ | $v_f(0)$ - $v_f(1)$ | > 1 (Since $n \geq 3$)

Therefore, the above labeling is not edge product cordial. Since this type of labeling gives maximum number of possible 1's to the edges, no other edge labeling can be edge product cordial.

Then, $S(C_n)$) does not admit edge product cordial labeling.

Hence, $S(C_n)$) is not an edge product cordial graph for n ≥ 3.

Illustration 2.5 : $S(C_5)$ is not an edge product cordial graph for n ≥ 3.



$S(C_n)$ has 2n vertices and 3n edges. Here, In $S(C_5)$, n = 5. The light and dark edges represents the edges with labels 1 and 0 respectively. The following type of labeling gives maximum number for $v_f(1)$. From the figure, it is clear that,

$e_f(1)$ = 2 +2 +2 +2 = 8 ; $e_f(0)$ = 5 + 2 = 7

Therefore, | $e_f(0)$ - $e_f(1)$ | = $|7 - 8|$ = 1

$v_f(1)$ = 4 ; $v_f(0)$ = 6

Thus, | $v_f(0)$ - $v_f(1)$ | = $|6 - 4| = 2 > 1$

Therefore, the above labeling is not edge product cordial. Since this type of labeling gives maximum number of possible 1's to the edges, no other edge labeling can be edge product cordial.

Hence, $S(C_5)$ is not an edge product cordial graph.

Theorem 2.6: For $n \geq 2$, S($P_n \odot K_1$) is an edge product cordial graph.

Proof: S($P_n \odot K_1$) has 4n vertices and 6n – 3 edges.

Let V($P_n \odot K_1$) = $\{v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_n\}$ where v_i's are the base path and u_i's are the end vertices. V(S($P_n \odot K_1$)) = $\{v_i, u_i, v_i', u_i' / i = 1 \text{ to } n\}$ where v_i' and u_i' are corresponding to v_i and u_i respectively.

Define $f : E(S(P_n \odot K_1)) \rightarrow \{0,1\}$ as follows.

Assign 1 to all the edges incident with u_i' for i = 1 to n and all the edges incident with u_i for i = 1 to n – 1 and to the edge $v_1'v_2$. Incident with each u_i , there are exactly 2 edges and incident with u_i' there are exactly one edge.

Therefore, $e_f(1) = n + 2(n-1) + 1 = 3n - 2 + 1 = 3n-1$

$e_f(0) = (6n - 3) - (3n - 1) = 3n - 2$

$\therefore |e_f(0) - e_f(1)| = 1$ ⟶ I

Correspondingly, the vertices u_i's for all i = 1 to n, the vertices u_i for i = 1 to n-1 and v_i' get the label 1.

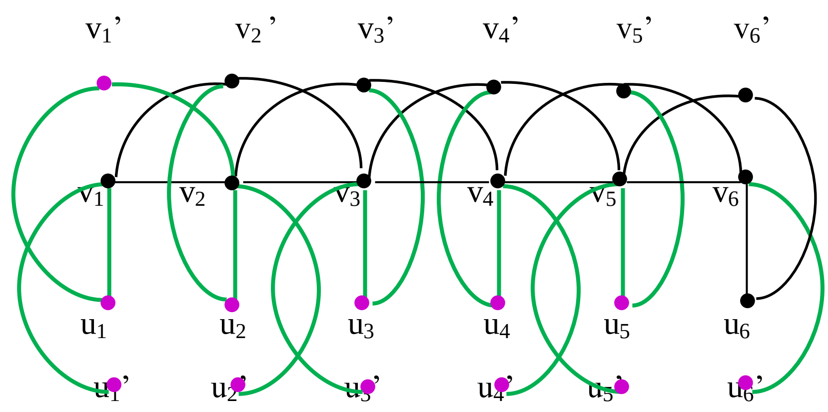
$\therefore v_f(1) = n + n - 1 + 1 = 2n$ and $v_f(0) = 4n - 2n = 2n$

$\therefore |v_f(0) - v_f(1)| = 0$ ⟶ II

By I and II, S($P_n \odot K_1$) admits edge product cordial labeling.

Hence, S($P_n \odot K_1$) is an edge product cordial graph.

Illustration 2.7: S($P_6 \odot K_1$) is an edge product cordial graph.



S($P_n \odot K_1$) has 4n vertices and 6n – 3 edges.

Here, In S($P_n \odot K_1$), n = 6. The light and dark edges represents the edges with labels 1 and 0 respectively.

$e_f(1) = 6 + 10 + 1 = 17$ and $e_f(0) = 2 + 9 + 5 = 16$.

Therefore, $|e_f(0) - e_f(1)| = |16 - 17| = 1 \leq 1$

$v_f(1) = 6 + 5 + 1 = 12$ and $v_f(0) = 6 + 5 + 1 = 12$

Thus, $| v_f(0) - v_f(1) | = |12 - 12| = 0 \leq 1$

Hence, S($P_6 \odot K_1$) is an edge product cordial graph.

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